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Miscellaneous Exercise (Set Theory)

Q.1. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, given an example.

(I) If $x \in A$ and $A \in B$, then $x \in B$ (II) If $A \subset B$ and $B \in C$, then $A \in C$. (III) If $A \subset B$ and $B \subset C$, then $A \subset C$

(IV) If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$

(V) If $x \in A$ and $A \not\subset B$, then $x \in B$

(VI) If $A \subset B$ and $x \notin B$, then $x \notin A$.

Soln (I) Let $A = \{2\}$, $B = \{\{2\}, 3\}$

$\Rightarrow 2 \in A$ and $A \in B$ but $2 \notin B$

So, $x \in A$ and $A \in B$ need not imply that $x \in B$.

Which is false.

(II) Let $A = \{2\}$, $B = \{2, 3\}$ and $C = \{\{2, 3\}, 4\}$

$\Rightarrow A \subset B$ and $B \in C$ but $A \notin C$

So, $A \subset B$ and $B \in C$ need not imply that $A \in C$.

Which is false.

(III) Let $1 \in A$, then

If $A \subset B \Rightarrow 1 \in B$ and if $B \subset C \Rightarrow 1 \in C$

$\Rightarrow 1 \in A \Rightarrow 1 \in C$.

Hence, $A \subset C$

So, if $A \subset B$ and $B \subset C$, then $A \subset C$

Which is true.

IV. Let $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{1, 3, 4\}$

$\Rightarrow A \not\subset B$ and $B \not\subset C$ but $A \subset C$

Hence, $A \not\subset B$ and $B \not\subset C$ need not imply that $A \not\subset C$.

Which is false.

V Let $A = \{1, 2\}$ and $B = \{2, 3, 4, 5\}$

It is clear that $1 \in A$ and $A \not\subset B$ but $1 \notin B$.

Hence, $x \in A$ and $A \not\subset B$ need not imply that $x \in B$.

and if $x \notin B \Rightarrow x \notin A$

Which is false.

(VI) Let $A \subset B$, then

if $x \in A \Rightarrow x \in B$

and if $x \notin B \Rightarrow x \notin A$.

Which is true.

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Q. 2. Let A, B and C be the sets such that $A\cup B = A\cup C$ and $A\cap B = A\cap C$.
Show that $B = C$.

Soln: It is given that, $A\cup B = A\cup C$

$\Rightarrow (A\cup B)\cap C = (A\cup C)\cap C$

$\Rightarrow (A\cap C)\cup(B\cap C) = C$ $[\because (A\cup C)\cap C = C]$

$\Rightarrow (A\cup B)\cup(B\cap C) = C$ $[\because A\cap C = A\cap B]$ —(I)

Again, $A\cup B = A\cup C$

$\Rightarrow (A\cup B)\cap B = (A\cup C)\cap B$

$\Rightarrow B = [(A\cap B)\cup(C\cap B)]$

$[\because (A\cup B)\cap B = B]$

$\Rightarrow B = (A\cap B)\cup(B\cap C)$ —— II

From I and II, we get $B = C$ Proved

Q. 3. Show that the following four conditions are equivalent:

(I) $A\subset B$ (II) $A - B = \phi$ (III) $A\cup B = B$ (IV) $A\cap B = A$.

Soln: (I) Since, $A\subset B \Leftrightarrow$ all the elements of A are in B

$\Leftrightarrow A - B = \phi$. Hence $I \Leftrightarrow (II)$ Proved.

(II) Now, we have $A - B = \phi$

$\Leftrightarrow A\subset B \Leftrightarrow A\cup B = B$

Hence, $II \Leftrightarrow (III)$ Proved.

(III) We have $A\cup B = B$

$\Leftrightarrow A\subset B \Leftrightarrow A\cap B = A$

$\therefore III \Leftrightarrow IV$ Proved

(IV) We have $A\cap B = A$

$\Leftrightarrow A\subset B$

$\therefore IV \Leftrightarrow I$ Proved.

Hence, $I \Leftrightarrow II \Leftrightarrow III \Leftrightarrow IV$

Proved